

Cosmology: Neutrinos also as one Kind of Dark Matter

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Abstract

In an effort to understand the large quantity of dark matter (25% versus 5% for the visible ordinary matter), we try to make the "family symmetry" as another gauge symmetry (and thus the source for another feeble interactions) and identify it as the main source of dark matter. One example is the $SU_c(3) \times SU(2) \times U(1) \times SU_f(3)$ extended Standard Model, but all species in the "visible ordinary matter", except the neutrinos, don't couple, except indirectly or only through higher order, to $SU_f(3)$ (the dark matter sector). In this note, we try to illustrate this aspect with an eye to detect the dark matter, through the detection of the neutrinos, the hypothetical species that couple directly to the dark-matter world. In other words, neutrinos may also be one kind of dark matter.

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1 Prelude

We know that the Standard Model is a gauge theory based on the gauge group $SU_c(3) \times SU_L(2) \times U(1)$ with the quark and lepton multiplets except the right-handed neutrinos. Thus, we might introduce the other $SU_f(3)$ with the right-handed neutrinos as the triplet - let's call it the family gauge group. Because so far the family gauge theory is completely independent of the Standard Model, including the multiplets used, we could form $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$, a trivial extension of the Standard Model. If we look at the quark multiplets in $SU_c(3)$ (in the construction), we could also replace the right-handed Dirac neutrinos by the whole Dirac neutrinos without much conceptional difficulty (with respect to the anomalies).

Thus, we have an extension[1] of the Standard Model that has three generations by explicit construction. If we introduce the Higgs mechanism in the family sector, we could make everything very massive - such that all the new family particles are not observed; hopefully barely observable such that we could verify the idea.

This offers an explanation why there are three generations - through the gauge-theory context. Otherwise, the three generation, or the famous Rabi's question of muon or lepton duplication, would be very mysterious.

We choose $(\nu_\tau, \nu_\mu, \nu_e)$ as the $SU_f(3)$ triplet, in order to realize the idea. Of course, one may choose the coupling to zero, or to be very small, such that one cannot detect

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the dark-matter particles at all. In this language, neutrinos are the only Standard-Model species that can be coupled directly to dark-matter particles. This coupling of neutrinos looks rather peculiar and it changes many things, though rather small, but to understand the family symmetry we should have this coupling or something similar.

Of course, there might be more than three families, but the question regarding the number of families would still be there. In fact, we assume that there is a gauge extension such as $SU_f(3)$ or others - based on the belief that any new interactions are described in terms of a gauge theory (just like the minimal Standard Model).

In this language, the $SU_f(3)$ gauge sector is the primary dark-matter world - while the neutrinos is something coupled to it. The $SU_f(3)$ gauge sector, or something similar, is used to characterize the "family symmetry" - in this context, the symmetry is visualized as the interactions of certain form.

2 Dirac Similarity Principle and Minimum Higgs Hypothesis

We could have another look at the minimal Standard Model - all the building blocks, such as electrons and quarks, can be described by the Dirac equations in certain forms and the search of the last forty years for the scalar fields such as Higgs still remains in vain. By construction, one may get other relativistic particles but it seems that point-like Dirac particles are already sufficient. Thus, we try to formulate the "Dirac Similarity Principle" and the "minimum Higgs hypothesis" as our working hypotheses [2] - particularly for the dark-matter world. These are working hypotheses, which are true so far for eighty years (Dirac) or for forty years and which could serve as good guidelines when a lot of unknowns (such as dark matter) are at stake.

We are living in a universe that at this moment there are a lot of unknowns - about 70% dark energy, 25% dark matter, and only 5% "visible" ordinary matter. Our minimum Standard Model of particle physics is used to describe the 5% "visible" ordinary matter, leaving 95% of the Universe untouched. Neutrinos, interacting so weakly with other ordinary matter, in some sense could be regarded as one kind of dark matter and in this paper be regarded as a messenger between the ordinary matter and the dark matter.

The story may be such that the Standard Model, which describes an immense amount of observing data, has been substantiated to high precision and it would be extended somehow to describe the bulk of dark matter. On the other hand, we all know that the 70% dark energy might be represented, to the first approximation, as due to the presence of the cosmological constant. Accordingly, we are left with the dark matter and the ordinary matter, the so-called "matter", to worry about.

There are some correlations between the dark-matter world and the "visible" ordinary-matter world - for instance, the Milky Way has about four or five times in mass of dark matter associated with it, judged from the rotation curve of the spiral. If there is no interaction, except the gravitation force, between the dark-matter particles and the ordinary-matter particles, then such correlations should not exist. On the other hand, if strong and electromagnetic interactions exist between the dark-matter particles and the ordinary-matter particles, it doesn't fit the description of the dark-matter galaxies or world. So, at best, it seems that the weak interactions could exist between the dark-matter particle and the ordinary-matter particle.

The fact that the electromagnetic and weak interactions are unified into the $SU_L(2) \times U(1)$ theory puts an important constraint - the dark-matter particles have to be singlets under $SU(2) \times U(1)$ or to be at most $(I_W, Y) = (0, 0)$ members (neutrinos). So, that dark-matter particles don't participate in strong and electromagnetic interactions put a severe constraint on the identity of the dark-matter particles - the only left-over in the ordinary-matter world would be neutrinos (and antineutrinos). That there is some galactic correlation as mentioned above indicates that neutrinos are also one kind of dark-matter particles - otherwise, there would be no correlation with the dark-matter world.

Could neutrinos be the messenger between the ordinary matter and the dark matter? Could neutrinos (in the ordinary-matter world) interact with the species in the dark-matter world? In fact, the neutrinos do not fit squarely into the minimum Standard Model - the Model says that they should be massless but the experiments tell not. In other words, the minimal Standard Model needs to be extended somehow. That is why we have proposed an extended Standard Model [1], that would make the neutrino sector much more interesting.

In other words, we think that "neutrinos are also one kind of dark matter" - neutrinos also interact with other dark-matter particles or neutrinos also have connections with dark-matter interactions. By this assertion, we rule out the dark-matter candidacy of the charge leptons, such as electrons, and of the quarks, thinking of these ordinary-matter particles that would be too visible. We suspect that there is indeed some bridge, such as neutrinos, between dark matter and ordinary matter, since it is believed that dark matter is clusterized near the visible world, such as the dark-matter galaxies.

There is a scenario for clustering - for ordinary matter, we know that they are clusterized into galaxies, clouds, etc. while for dark matter they might be clusterized into invisible galaxies, clouds, etc., maybe of order ten or larger (in length). Neutrinos have tiny mass in the sub-eV range or the feeble interactions effectively of the sub-eV range - it fits the description. This in fact may explain why our Milk Way or the other galaxies has a large spiral arm. So, the invisible dark-matter galaxies, of size 10^3 or bigger, serve as the hosts of the ordinary galaxies, such as the Milk Way.

Dirac invented the so-called "Dirac equation" to describe the electron, which later turns out to be the first point-like Dirac particle (in the Standard Model). For the last eighty years, our searches for point-like Dirac particles could be summarized by the Standard Model; in fact, in our world, the point-like Dirac particles belong only to the world of the Standard Model; and in our space-time they are described as "quantized Dirac fields". On the other hand, for the last forty years, we were looking for "Higgs particles", spin-zero quantized Klein-Gordon fields, but surprisingly enough nothing so far. Therefore, I suggest that we could formulate our forty-year experience into the "minimum Higgs hypothesis". In theoretician's search for the new models, "Dirac similarity principle" and the "minimum Higgs hypothesis" greatly simplifies the scope. Thinking of 25% dark matter, the two empirical working hypotheses should help considerably. (One is the experience of the last eighty years - what are "point-like Dirac particles" and are there other point-like configurations that exist in our space-time? The other is that of the last forty years - where are the Higgs particles?)

The another consideration related to the theory of dark matter is the so-called "symmetry", including the super-symmetry. In fact, we have a lot of rooms or loopholes in this regard, but the symmetry considerations should play a major role in the theory of

dark matter. Maybe the "symmetry" is equivalent to the "interactions" of some form. To this end, we use "Dirac similarity principle" and the "minimum Higgs hypothesis" as two simplifying working conjectures.

Under "Dirac similarity principle" and the "minimum Higgs hypothesis", the extended Standard Model would be unique if the gauge group is fixed. For example, the extra Z'^0 model, for the $SU_L(2) \times U(1) \times U(1)$ gauge group, is unique [3]. The group $SU_L(2) \times SU_R(2) \times U(1)$ now gives rise to a unique left-right symmetric model - one out of the left-right models [4]. In fact, some symmetries may eventually be gauged and so carry feeble interactions (and could be classified as "dark-matter" interactions). In the rest of the paper, we would focus most of our discussions about the family symmetry.

We have been curious why there are three generations of fermions, i.e., quarks, charged leptons, and neutrinos; the so-called "family symmetry" but without giving a reason. To our knowledge, these particles are all described by Dirac equation and the so-called point-like Dirac particles (or, quantized Dirac fields). The similarity to the electron is rather strange and thus I call it "Dirac similarity principle", although its applicability to the case of neutrinos is waiting for verification. These might be only point-like Dirac particles realized in our space-time (as described by quantized Dirac fields).

Quarks and charged leptons are charged and too much "visible" - thus, they are better be "family-blind", as we promote "family symmetry" to the "family gauge symmetry". As said earlier, the three neutrinos (ν_τ, ν_μ, ν_e) could be a natural suspect that bridges the "visible" ordinary matter world to the dark matter world.

In fact, we suspect that there are symmetries which may be dynamical but the involved interactions may be too feeble to be observed. This applies to not only the family symmetry but also some other symmetries. In this sense, we suspect that there is some dynamical "family gauge symmetry" rather than the simply "family symmetry", except that the family particles connect the ordinary world too weakly and thus could not be easily seen. Well, if it is the interaction related to the family symmetry, then it'd better be the gauge family symmetry - the other would be mediated by the (scalar) Higgs fields but it happens very rarely; we just look at what has happened for the minimum Standard Model.

The other clue that neutrinos may play a "bridging" role comes from the fact that neutrinos are massive and oscillating while in the minimum Standard Model they should be massless. One way to save the situation is that there are dark-matter Higgs scalar fields, to which neutrinos couple in order to generate their tiny masses in a renormalizable way.

In this way, the massive $SU_f(3)$ is emerging from the thinking. [I think that it could be a different group - since $SU_f(3)$ sounds too trivial.] Let us call the gauge bosons "familons". We have to use two complex scalar field triplets under $SU_f(3)$ in order to make all familons massive. We assume that (ν_τ, ν_μ, ν_e) would be the only triplet from "visible" ordinary matter sector. For simplicity, We could call the model as "the minimum family standard model (mfSM)" [1]. From now on, the familons and the related Higgs particles are called as "dark-matter particles" and only neutrinos play the role of the bridging particles.

At this point, we could call the extension of the Standard Model through the gauging of the family symmetry as a "natural" way - since it preserves all the symmetries. Of course, whether the family symmetry can be gauged in some way should be answered experimentally. If not, we should try some sort of the "see-saw" mechanisms[5] for neutrinos although the neutrinos remain to play the bridging role between the dark matter and the

ordinary matter. In this case, there is of course violation of "Dirac similarity principle".

3 Neutrinos in the Bridging Role

Because of the known three families, we introduce $(\nu_\tau, \nu_\mu, \nu_e)$ as the $SU_f(3)$ fundamental triplet. The three neutrinos serve to label the three generations/families. Whether or not we should choose $SU(3)$ as the family group (a trivial choice) is a subtle question. This question should be further pursued, in particular since it might take time to identify what feeble interactions the family symmetry corresponds to.

Returning to our model, i.e., the so-called mfSM[1], the neutrino triplet $\Psi(x) (\equiv (\nu_\tau, \nu_\mu, \nu_e))$ involves interactions of two kinds:

(1) Neutrino-familon interactions:

$$-\kappa \bar{\Psi}(x) \frac{\lambda^a}{2} F_\mu^a(x) \Psi(x); \quad (1)$$

Here Ψ is the neutrino triplet while F_μ^a the eight family gauge fields.

(2) Neutrino-Higgs (mass) interactions:

$$i\eta \bar{\Psi} \times (\Phi_+ + \epsilon \Phi_-) \cdot \Psi. \quad (2)$$

Here Φ_\pm are the triplet Higgs fields which make all familons massive.

These two couplings are the only couplings that are renormalizable. As for the couplings, we do not know that whether they are all real or some of them are complex. The above interactions of two types, presumably fairly weak, occur in the neutrino sector and differentiate neutrinos - so violate the $\tau - \mu - e$ universality.

If the family gauge group is $SU(3)$ as in [1], the eight gauge bosons F_μ^a and the pair of the family Higgs triplets Φ_\pm , together with the three neutrinos (and anti-neutrinos), would serve the main body of the dark-matter particles. Except the known neutrinos, these unknown (family) particles are introduced to be very massive, presumably greater than $1 TeV$ - we are thinking of what could be seen at the Large Hadron Collider (LHC).

In fact, if we could introduce the right-hand neutrinos as $SU_L(2)$ singlets and the standard Higgs doublet φ , we could generate the neutrino mass by the coupling $\bar{L}\nu_R\varphi^*$. This is the only place that ν_R appears. The masses from the top quark till the neutrinos all come from φ - it covers too wide the range from $10^{12}eV$ (the top quark mass) to $10^{-3}eV$ (the neutrino mass), 15 orders of magnitude. I would suggest something much more plausible - either Eqs. (2) and (1) as in mfSM[1] or that the couplings should be proportional to $(v/v')^2$ where v (v') is the vacuum expectation value of the standard Higgs (the remote Higgs). Thus, the tiny neutrino masses have to come somewhere else and the masses of quarks and charged leptons could all come from the standard Higgs doublet. The coupling size should vary with the distance of the generation to which the coupling occurs - just like the case of the CKM matrix elements. For the sake of simplicity, we could call it "law of the coupling varying with the distance" or "law of coupling". Note that this law should apply to both couplings in the mfSM, Eqs. (1) and (2), since the familons or the associated Higgs would automatically be classified "remote Higgs" (as against the standard Higgs doublet in the Standard Model).

We could "formulate" our working conjecture about the "minimum Higgs hypothesis" as follows: There should be the minimum number of Higgs multiplets and the couplings to the "remote" Higgs should be much "smaller" compared to the leading Higgs (such as the Standard Higgs doublet).

Alternatively, if right-handed neutrinos are there but the neutrino mass-generation mechanism does not use the Standard-Model Higgs, this would pose as the basic problem. In general, in a given extended Standard Model, each particle know the rest unless some rules forbid that - this is why we have the "minimum Higgs hypothesis".

If we should introduce $SU_f(3)$ into the game, these gauge bosons and the related Higgs particles have to be massive if the neutrinos "know" the existence of these particles. That is, the dark Higgs mechanisms are there to protect why the Standard Model are so good. In other words, if dark matter particles are there and if they communicate with neutrinos, then the validity of the Standard Model should somehow be protected.

In any event, these interactions, plus the dark Higgs mechanisms, violate a lot of symmetries, such as the $\tau - \mu - e$ universal symmetry, but the symmetry-violating part is in general very small and, because they are dark, these might not be seen easily.

On the neutrino masses in the mfSM, we have both the neutrino-Higgs mass interactions and "radiative" corrections from neutrino-familon loops. The combined analysis of the two effects is required to get the real understanding of the experimental data.

As said earlier, all the familons or dark Higgs should not be massless or too light. For example, the cross sections for neutrino scatterings (off quarks or off charged leptons) would be modified significantly by their presence (if some of these dark species are massless or small), so as to deviate from the known Standard Model results[6]. In other words, any of these masses will be taken to be $\geq 1\text{ TeV}$, to be safe. Of course, it would be much more fun if these dark matter particles would be observed some day.

The other effect is the familon-loop corrections in the scattering of neutrinos off the electron or off the quarks - similar to the so-called ρ -parameter calculation in the Standard Model[6]. Although the ρ -parameters for the τ - or μ -sectors are completely unknown - and maybe remain to be completely unknown forever, the breakdown of the $\tau - \mu - e$ universality is a genuine possibility. On the other hand, the ρ -parameter in the electron case, in view of its relative larger error, does permit for a larger domain for the familon masses.

Maybe we need to emphasize that the violation in the neutrino sector of the $e - \mu - \tau$ universality, the difference between $\nu - e$ and $\nu - \mu$ scatterings as caused by familon-loops, and the different ρ parameters, all offers the test of the family gauge idea. These should also test the nature of the dark-matter world.

Maybe the most important issue has to do with Cosmology. Question No.1 has to do with whether a phase transition may occur at 10^{-12} or 10^{-13} sec, or slightly earlier, after the Big Bang. Here we have used 1 TeV as the range for familon masses. If so, what is the major effect of the familon phase transition? Question No.2 has to do with the fact that all familons and all family Higgs should eventually decay into lighter species (such as neutrinos in the mfSM) - is this some observable effect with the cosmological time as a measuring stick? These questions are difficult to answer quantitatively in the mfSM as of now since our experimental understanding of dark matter is lacking.

4 Neutrinos Do Oscillate!!

Neutrinos do oscillate, according to the bulk of experimental data in recent years!! The mass eigenstates are different from the flavor eigenstates. Maybe the first important issue is to check out how they oscillate - do they, $(\nu_e, \nu_\mu, \nu_\tau)$, oscillate among themselves? Or, into something else? In what follows, we assume that they oscillate among themselves, i.e. $\Delta L = 0$, for simplicity.

Neutrinos enter weak interactions via the flavor eigen-states while the mass terms through the mass eigen-states. The known heaviest mass[6] is about $0.04 eV < m_i < (0.2 - 0.4) eV$. This is rather interesting for many reasons. For example, in the reaction $\nu + \bar{\nu}_{CB} \rightarrow e^- + e^+$ by high energy ν , the CB neutrinos now have the rest mass much bigger than the temperature ($1.9^\circ K$), maybe making the detection of cosmic background neutrinos somewhat easier.

For the $\Delta L = 0$ oscillation, the formulae read[7]

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}\{U_{\alpha i}^* U_{\beta i} U_{\beta j} U_{\alpha j}^*\} \sin^2[1.27 \Delta m_{ij}^2 (L/E)] \\ + 2 \sum_{i>j} \text{Im}\{U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*\} \sin[2.54 \Delta m_{ij}^2 (L/E)]. \quad (3)$$

Here α, β flavor indices, i, j the mass eigenstates, $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ is in eV^2 , L is in km , and E is in GeV .

For the 8B solar neutrinos, we have $E_\nu^{max} = 14.06 MeV$ and $\Delta m_{i,j} = 10^{-5} \rightarrow 10^{-3} eV^2$ so that the distance L would be $(10 \rightarrow 1,000) km$ to guarantee the argument of sine or cosine would be order unity. The other golden number is the cross section of the order $10^{-42} cm^2$, which is rather small. Thus, it is rather difficult to detect neutrino oscillations for MeV neutrinos, in fact, for all neutrinos.

The above formulae, if neutrinos oscillate into themselves, could help us to design certain experiments or to analyze the feasibility of a given experiment or a certain reaction.

One very important issue is that if neutrinos are massive then these neutrinos would tend to clusterize, maybe around the galaxies. If clusterized around the galaxies, the CB neutrino distributions would deviate from the uniform distributions like in the CMB photons. As an estimate, we may assume that the neutrinos may clusterize as around the galaxy but not farther than the parent galaxies. So, in a rich galaxies region such as our Virgo cluster, the galaxy region should occupy only up to one part in 10^5 (just an estimate). So, the neutrino density near the galaxy would be enhanced compared to the average density by a factor of 10^5 or higher.

What if the gauge-family symmetry is there [1], neutrino should still oscillate. But it would be a lot more interesting. Imagine some world that is dark with which only neutrinos could couple; our Universe is quite unthinkable.

5 Prehistory to Electroweak Phase Transition

Let us look at the Universe at time $t = 10^{-12} sec$. The calculation, using the Einstein General-Relativistic equation in the Freedman-Robertson-Walker metric and the standard Equation of States (EOS), yields

$$T \approx 1 TeV, \quad \rho_\gamma \approx 6.4 \times 10^{24} gm/cm^3, \quad \rho_m \approx 10^{14} gm/cm^3. \quad (4)$$

Analogously, we have, at $t = 10^{-13} \text{sec}$,

$$T \approx 3.2 \text{TeV}, \quad \rho_\gamma \approx 6.4 \times 10^{26} \text{gm/cm}^3, \quad \rho_m \approx 3.2 \times 10^{15} \text{gm/cm}^3. \quad (5)$$

The atomic scale is about $1.6726 \times 10^{-24} \text{gm}/(0.529 \times 10^{-8} \text{cm})^3$ (proton mass/(Bohr radius)**3) or 11.30gm/cm^3 . The atomic scale is where quantum physics is all about. The above numbers are comparable to the nuclear matter density $2 \times 10^{14} \text{gm/cm}^3$.

Somewhere in the region, we have the $SU_f(3)$ phase transition. In a $SU(3)$ typical phase transition, we have the so-called vacuum change with ρ_{vac}^f , which we may choose a few TeV for simplicity. Before the familon phase transition, we refer the vacuum as the "pre-familon vacuum"; after, the "post-familon vacuum". For further discussions on phase transition, one may consult Hwang and Kim[8].

People know a little nuclear physics would start worry about those densities: A star of a solar mass would collapse into a black hole at about $\rho_m \approx 10^{16} \text{gm/cm}^3$ but with much lower temperatures. How about the photon sphere with much higher densities, that is greater than $\rho_\gamma \approx 10^{24} \text{gm/cm}^3$? Questions of this kind may in fact lead to some big revolution eventually.

6 Neutrinos are also one kind of dark matter.

In the extended Standard Model with the gauge family built-in [1], particle physics in the neutrino sector is expected to be different. The dark-matter interactions are now much weaker than, e.g., weak interactions and symmetry breaking effects might be very common with these new interactions. With the neutrinos as the bridge between the dark matter and ordinary visible matter, we could study neutrinos as the means of probing dark matter further.

For example, the universality among $\tau - \mu - e$ is broken from the mass, from the coupling to the dark Higgs $\nu_1 - \nu_2 - \phi$, and even from the neutrino-familon vertex $\bar{\Psi} F_\mu \gamma^\mu \Psi$, noting that the family $SU_f(3)$ is spontaneously broken. Maybe this would be the first experimental evidence of the familons, that we could try to establish.

On the other hand, we should work out a practical example: neutrino-electron scattering $\nu + e \rightarrow \nu + e$. The reason that the dark-matter particles, including the familons and the dark Higgs, cannot be massless is due to absence of $\nu + e \rightarrow \nu + e + \text{dark particle}$, which would otherwise deviate from the Standard Model. In addition, the loop corrections, due to dark particles, in $\nu + e \rightarrow \nu + e$ cannot be too big so that the agreement between the Standard Model and the experiments would be violated. On the hand, someday we could try to measure $\nu + \mu \rightarrow \nu + \mu$ to test the violation of the $\tau - \mu - e$ universality. All in all, we should focus our attention in studying potential couplings between the dark sector and the standard model - the neutrinos would be likely candidates.

Of course, it seems that none of these experiments could be easy. But we should try to do some of them since even though our suggested model might not be there but all the competing models have rather similar characteristics. Our progress depends on how to identify the most important experiment(s). After all, remember that neutrinos are also one kind of dark matter.

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